Electric Quadrupole Transitions in Odd-Mass Spherical Nuclei*

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(Received 6 September 1963)

The wave functions derived by Kisslinger and Sorensen from the pairing plus quadrupole force model for atomic nuclei are used to compute theoretical $E2$ electromagnetic transition rates between various low-lying states in odd-mass spherical nuclei from Ni to Pb. Comparison is made with experimental data where available. The agreement between theory and experiment is quite good, a large majority of the forty or so cases agreeing within a factor of 2 while the data cover a range of more than a factor of 1000.

I. INTRODUCTION

THE occurrence of giant quadrupole effects in nuclei has been known for a long time.[1] For the deformed nuclei, one observes ground-state quadrupole moments which are many times the single-particle magnitude, and also $E2$ transition rates which are many times enhanced above the Weisskopf estimate. For spherical nuclei one observes $E2$ transitions from the $2+$ to $0+$ ground state of the even systems which are enhanced from a few times to one hundred times the single-particle rate. It has been shown that all these effects as well as "single particle" phenomena can be explained in considerable detail by a nuclear model[2] in which particles moving in a spherical potential well interact with a pairing plus quadrupole force. It is the purpose of this note to demonstrate that this model also agrees in considerable detail with the presently available data on $E2$ transition rates in odd-mass spherical nuclei. In addition, an extensive table of theoretical $E2$ rates for these nuclei is included to suggest possible interesting cases for future experimental study.

II. CALCULATION

With the use of the approximations used by Kisslinger and Sorensen[3,4] to treat the pairing plus quadrupole Hamiltonian for spherical nuclei, the nuclear states are characterized by two types of excitations, quasiparticles and phonons. For an even-even nucleus, the lowest excited state is the one-phonon $2+$ state, which, because of the energy gap for quasiparticle excitations, is well separated from them and may be treated alone. Good agreement with the $2+$ energy and transition rate to the ground state is obtained with the use of a pairing and a quadrupole strength parameter which are smooth functions of mass number, with the exception that the calculated energy is too low, and the $B(E2)$ value too large for nuclei very near a region of deformation. To reduce this difficulty, for the calculation of the properties of odd-mass nuclei, the quadrupole coupling strength was chosen in Ref. 4 to fit the $2+$ energy of the adjacent even-even nuclei.

For odd nuclei, many states of one quasiparticle and zero, one, or two phonons will lie rather close in energy, and thus must not be treated as independent excitations. In Ref. 4, the pairing plus quadrupole Hamiltonian is approximately diagonalized in the space of states containing one quasiparticle and up to two phonons. The approximation is to retain in the Hamiltonian only the terms which scatter the quasiparticle while at the same time creating or destroying a phonon. The no-phonon to one-phonon matrix elements are

$$\langle 0|\alpha_j|H_{\text{int}}|(B^\dagger\alpha_{j'}^\dagger)j|0\rangle = -\bar{\chi}(5/4\pi)^{\frac{1}{2}} \times\langle j|r^2|j'\rangle C_0{}^2{}_{\frac{1}{2}}{}^j{}_{\frac{1}{2}}{}^{j'}(-1)^{j-j'}(U_jU_{j'}-V_jV_{j'}), \quad (1)$$

where the effective coupling constant $\bar{\chi}$, defined in Ref. 4, depends on the energy of the adjacent even nuclei as described. The creation operators $B^\dagger$ and $\alpha_j{}^\dagger$ create $2+$ phonons and j-type quasiparticles, respectively, where j represents the angular momentum (and parity) of the shell model state. The state $|0\rangle$ is the vacuum for quasiparticles and phonons and represents the ground state of an even-even nucleus. The quantities U and V are the usual occupation factors of the pairing theory. The one-phonon to two-phonon matrix element, given in Ref. 4 contains the same factor of $(U_jU_{j'}-V_jV_{j'})$.

The wave functions resulting from the diagonalization procedure are of the form

$$\psi_j = C_{j00}{}^j\alpha_j{}^\dagger|0\rangle + \sum_{j'} C_{j'12}{}^j(B^\dagger\alpha_{j'}{}^\dagger)_j|0\rangle + \cdots. \quad (2)$$

The C coefficients [not to be confused with the Clebsch Gordan coefficient of Eq. (1)] for the lowest few states of the spherical nuclei are computed and tabulated in Ref. 4. The sum on j' is over all single-particle states of the same parity as j for which $|j-j'|\leq 2$. The parenthesis indicates that the $2+$ phonon and j' quasiparticle are coupled to an angular momentum j. Owing to the presence of the U, V factor in Eq. (1), the

* Work supported, in part, by the U. S. Office of Naval Research and by the U. S. Atomic Energy Commission at Brookhaven National Laboratory, where part of this work was performed, Summer, 1963.

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TABLE I. Reduced $E2$ transition probabilities for odd mass spherical nuclei. The first and second column lists the isotope, and the levels between which the transition occurs. Columns 3, 4, and 5, list the single particle [Eq. (7)], theoretical, [Eq. (6)], and experimental [$B(E2)$], values divided by $2j_f+1$, where j_f is the final angular momentum, in units of $10^{-50}e^2$.

Isotope	Transition	$B(E2)_{\mathrm{s.p.}}/2j_f+1$	$B(E2)_{\mathrm{theor}}/2j_f+1$	$B(E2)_{\mathrm{exptl.}}/2j_f+1$
$_{28}$Ni^{59}	$p_{3/2}\, f_{5/2}$	0.010	0.038	
Ni^{61}	$p_{3/2}\, f_{5/2}$	0.010	0.0035	0.012[a]
Ni^{63}	$p_{3/2}\, f_{5/2}$	0.011	0.036	
$_{30}$Zn^{63}	$p_{3/2}\, f_{5/2}$	0.011	0.0002	
Zn^{65}	$p_{3/2}\, f_{5/2}$	0.011	0.078	
Zn^{67}	$p_{3/2}\, f_{5/2}$	0.012	0.105	0.52[a]
	$p_{1/2}\, f_{5/2}$			0.01[a]
$_{37}$Rb^{85}	$p_{3/2}\, f_{5/2}$	0.016	0.148	0.077[b]
Rb^{87}	$p_{3/2}\, f_{5/2}$	0.017	0.122	0.12[b]
$_{48}$Cd^{107}	$s_{1/2}\, d_{5/2}$	0.15	0.023	
	$d_{5/2}\, g_{7/2}$	0.007	0.0065	
Cd^{109}	$s_{1/2}\, d_{5/2}$	0.15	0.81	
	$d_{3/2} g_{7/2}$	0.10	0.72	
	$d_{3/2}\, d_{5/2}$	0.022	0.012	
	$s_{1/2}\, d_{3/2}$	0.15	1.98	
	$d_{5/2} g_{7/2}$	0.007	0.051	
Cd^{111}	$s_{1/2}\, d_{5/2}$	0.16	1.78	2.4[c]
	$d_{3/2}\, g_{7/2}$	0.10	0.51	
	$d_{3/2}\, d_{5/2}$	0.023	0.0032	
	$s_{1/2}\, d_{3/2}$	0.16	2.09	2.7[c]
	$d_{5/2}\, g_{7/2}$	0.008	0.079	
Cd^{113}	$s_{1/2}\, d_{5/2}$	0.16	2.48	5.0[c]
	$d_{3/2}\, g_{7/2}$	0.10	0.212	
	$d_{3/2}\, d_{5/2}$	0.023	0.0004	
	$s_{1/2}\, d_{3/2}$	0.16	2.02	2.7[c]
	$d_{5/2}\, g_{7/2}$	0.008	0.063	
Cd^{115}	$s_{1/2}\, d_{5/2}$	0.17	2.90	
	$d_{3/2}\, g_{7/2}$	0.11	0.021	
	$d_{3/2}\, d_{5/2}$	0.024	0.0007	
	$s_{1/2}\, d_{3/2}$	0.17	1.43	
	$d_{5/2}\, g_{7/2}$	0.008	0.049	
$_{50}$Sn^{117}	$s_{1/2}\, d_{3/2}$	0.17	0.134	0.02–0.10[d]
Sn^{119}	$s_{1/2}\, d_{3/2}$	0.17	0.0001	<0.2[d]
Sn^{121}	$s_{1/2}\, d_{3/2}$	0.18	0.111	
Sn^{123}	$s_{1/2}\, d_{3/2}$	0.18	0.393	
Sn^{125}	$s_{1/2}\, d_{3/2}$	0.19	0.61	
$_{51}$Sb^{115}	$s_{1/2}\, d_{5/2}$	0.17	1.34	
	$d_{5/2}\, g_{7/2}$	0.008	0.154	
Sb^{117}	$s_{1/2}\, d_{5/2}$	0.17	1.30	
	$d_{5/2}\, g_{7/2}$	0.008	0.156	
Sb^{119}	$s_{1/2}\, d_{5/2}$	0.18	1.31	
	$d_{5/2}\, g_{7/2}$	0.008	0.155	
Sb^{121}	$s_{1/2}\, d_{5/2}$	0.18	1.42	1.7[e]
	$d_{5/2}\, g_{7/2}$	0.009	0.168	
	$d_{3/2}\, g_{7/2}$	0.11	0.98	1.04[e]
	$d_{3/2}\, d_{5/2}$	0.026	0.141	1.1[e]
	$s_{1/2}\, d_{3/2}$	0.18	0.134	
Sb^{123}	$s_{1/2}\, d_{5/2}$	0.18	1.16	
	$d_{5/2}\, g_{7/2}$	0.009	0.146	0.065[f]
Sb^{125}	$s_{1/2}\, d_{5/2}$	0.19	1.33	
	$d_{5/2}\, g_{7/2}$	0.009	0.168	
$_{52}$Te^{121}	$s_{1/2}\, d_{3/2}$	0.18	0.174	5.4[d]
Te^{123}	$s_{1/2}\, d_{3/2}$	0.18	0.86	0.45–0.8[d]
Te^{125}	$s_{1/2}\, d_{3/2}$	0.19	1.40	
$_{53}$I^{125}	$s_{1/2}\, d_{5/2}$	0.19	0.96	
	$d_{5/2}\, g_{7/2}$	0.009	0.134	
I^{127}	$s_{1/2}\, d_{5/2}$	0.19	1.09	~1.0[g]
	$d_{5/2}\, g_{7/2}$	0.009	0.156	
I^{129}	$s_{1/2}\, d_{5/2}$	0.20	1.09	
	$d_{5/2}\, g_{7/2}$	0.009	0.134	
I^{131}	$s_{1/2}\, d_{5/2}$	0.20	1.03	
	$d_{5/2}\, g_{7/2}$	0.009	0.111	
$_{54}$Xe^{127}	$s_{1/2}\, d_{3/2}$	0.19	1.96	
Xe^{129}	$s_{1/2}\, d_{3/2}$	0.20	1.87	
Xe^{131}	$s_{1/2}\, d_{3/2}$	0.20	1.58	
Xe^{133}	$s_{1/2}\, d_{3/2}$	0.20	1.32	
$_{55}$Cs^{129}	$s_{1/2}\, d_{5/2}$	0.20	1.25	
	$d_{5/2}\, g_{7/2}$	0.009	0.229	
Cs^{131}	$s_{1/2}\, d_{5/2}$	0.20	1.41	
	$d_{5/2}\, g_{7/2}$	0.009	0.174	
Cs^{133}	$s_{1/2}\, d_{5/2}$	0.20	1.48	
	$d_{5/2}\, g_{7/2}$	0.010	0.097	0.20[h]
Cs^{135}	$s_{1/2}\, d_{5/2}$	0.21	1.30	
	$d_{5/2}\, g_{7/2}$	0.010	0.043	
Cs^{137}	$s_{1/2}\, d_{5/2}$	0.21	1.24	
	$d_{5/2}\, g_{7/2}$	0.010	0.010	
$_{56}$Ba^{131}	$s_{1/2}\, d_{3/2}$	0.20	2.75	
Ba^{133}	$s_{1/2}\, d_{3/2}$	0.20	2.20	
Ba^{135}	$s_{1/2}\, d_{3/2}$	0.21	2.17	0.24[f]
Ba^{137}	$s_{1/2}\, d_{3/2}$	0.21	1.60	0.65[f]
$_{57}$La^{137}	$s_{1/2}\, d_{5/2}$	0.21	1.47	
	$d_{5/2}\, g_{7/2}$	0.010	0.0067	
La^{139}	$s_{1/2}\, d_{5/2}$	0.22	1.41	
	$d_{5/2}\, g_{7/2}$	0.010	0.0014	
$_{59}$Pr^{141}	$s_{1/2}\, d_{5/2}$	0.22	1.50	
	$d_{5/2}\, g_{7/2}$	0.010	0.0008	<0.04[a]
Pr^{143}	$d_{5/2}\, g_{7/2}$	0.011	0.0098	
$_{60}$Nd^{145}	$p_{3/2}\, f_{7/2}$	0.15	0.78	1.7[i]
Nd^{147}	$p_{3/2}\, f_{7/2}$	0.15	1.09	
$_{61}$Pm^{145}	$d_{5/2}\, g_{7/2}$	0.011	0.0088	
Pm^{147}	$d_{5/2}\, g_{7/2}$	0.011	0.090	0.36[j]
Pm^{149}	$d_{5/2}\, g_{7/2}$	0.011	0.039	
$_{62}$Sm^{147}	$p_{3/2}\, f_{7/2}$	0.15	0.89	
Sm^{149}	$p_{3/2}\, f_{7/2}$	0.15	1.20	
$_{77}$Ir^{191}	$s_{1/2}\, d_{3/2}$	0.32	11.4	
	$d_{3/2}\, d_{5/2}$	0.047	7.35	10.5[k]
Ir^{193}	$s_{1/2}\, d_{3/2}$	0.33	9.15	
	$d_{3/2}\, d_{5/2}$	0.048	6.40	12.5[c]
$_{78}$Pt^{193}	$p_{1/2}\, f_{5/2}$	0.33	5.15	
	$p_{1/2}\, p_{3/2}$	0.33	4.95	
	$p_{3/2}\, f_{5/2}$	0.048	0.305	
Pt^{195}	$p_{1/2}\, f_{5/2}$	0.34	4.95	5.0[l]
	$p_{1/2}\, p_{3/2}$	0.34	2.78	4.5[l]
	$p_{3/2}\, f_{5/2}$	0.048	0.0005	
Pt^{197}	$p_{1/2}\, f_{5/2}$	0.34	3.50	
	$p_{1/2}\, p_{3/2}$	0.34	0.73	
	$p_{3/2}\, f_{5/2}$	0.049	0.296	
$_{79}$Au^{195}	$d_{3/2}\, g_{7/2}$	0.22	6.55	
	$s_{1/2}\, d_{3/2}$	0.33	1.36	3.2[m]
	$d_{3/2}\, d_{5/2}$	0.048	5.8	
Au^{197}	$d_{3/2}\, g_{7/2}$	0.22	4.40	5.4[k]
	$s_{1/2}\, d_{3/2}$	0.33	0.74	2.7[m]
	$d_{3/2}\, d_{5/2}$	0.048	3.8	5.6[k]
Au^{199}	$d_{3/2}\, g_{7/2}$	0.22	4.20	
	$s_{1/2}\, d_{3/2}$	0.34	0.55	
	$d_{3/2}\, d_{5/2}$	0.049	3.6	
$_{80}$Hg^{195}	$p_{1/2}\, f_{5/2}$	0.34	4.90	11.5[n]
	$p_{1/2}\, p_{3/2}$	0.34	4.35	
	$p_{3/2}\, f_{5/2}$	0.049	0.177	
Hg^{197}	$p_{1/2}\, f_{5/2}$	0.34	3.53	3.5[n]
	$p_{1/2}\, p_{3/2}$	0.34	1.76	
	$p_{3/2}\, f_{5/2}$	0.049	0.0	
Hg^{199}	$p_{1/2}\, f_{5/2}$	0.35	2.64	6.3[n]
	$p_{1/2}\, p_{3/2}$	0.35	0.44	2.5[k]
	$p_{3/2}\, f_{5/2}$	0.050	0.194	
Hg^{201}	$p_{1/2}\, f_{5/2}$	0.35	0.45	
	$p_{1/2}\, p_{3/2}$	0.35	0.042	
	$p_{3/2}\, f_{5/2}$	0.050	0.424	

TABLE I (*Continued*).

Isotope	Transition	$B(E2)_{\mathrm{s.p.}}/2j_f+1$	$B(E2)_{\mathrm{theor}}/2j_f+1$	$B(E2)_{\mathrm{exptl.}}/2j_f+1$
$_{80}Hg^{203}$	$p_{1/2}\,f_{5/2}$	0.36	0.208	
	$p_{1/2}\,p_{3/2}$	0.36	0.89	
	$p_{3/2}\,f_{5/2}$	0.051	0.407	
$_{81}Tl^{199}$	$s_{1/2}\,d_{3/2}$	0.35	2.74	
	$d_{3/2}\,d_{5/2}$	0.050	0.025	
Tl^{201}	$s_{1/2}\,d_{3/2}$	0.35	2.87	
	$d_{3/2}\,d_{5/2}$	0.050	0.014	
Tl^{203}	$s_{1/2}\,d_{3/2}$	0.36	2.45	3.1[c]
	$d_{3/2}\,d_{5/2}$	0.051	0.005	<0.3[c]
	$s_{1/2}\,d_{5/2}$	0.36	4.08	3.5[c]
Tl^{205}	$s_{1/2}\,d_{3/2}$	0.36	2.04	2.5[c]
	$d_{3/2}\,d_{5/2}$	0.052	0.0	<0.15[c]
	$s_{1/2}\,d_{5/2}$	0.36	3.73	1.9[c]
$_{82}Pb^{203}$	$p_{1/2}\,f_{5/2}$	0.36	$\sim$0.01	0.13[n]
	$p_{3/2}\,f_{5/2}$	0.051	0.039	
Pb^{205}	$p_{1/2}\,f_{5/2}$	0.36	0.016	
	$p_{3/2}\,f_{5/2}$	0.052	0.066	
	$p_{1/2}\,p_{3/2}$	0.36	0.074	
Pb^{207}	$p_{1/2}\,f_{5/2}$	0.37	0.37	0.63[n]
	$p_{3/2}\,f_{5/2}$	0.052	0.052	
	$p_{1/2}\,p_{3/2}$	0.36	0.36	

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coupling is often relatively weak for the ground state, for which the factor may be small. For most other low-lying states, including some ground states, the coupling is strong enough that $C_{j'12}{}^{j}$ is a sizeable fraction of $C_{j00}{}^{j}$. It is for this reason that $E2$ transitions in odd nuclei are often much more enhanced than the corresponding ground-state quadrupole moments.

The $E2$ transition operator contains two terms[4]

$$\mathfrak{M}(E2)=\mathfrak{M}(E2)_{\mathrm{s.p.}}+\mathfrak{M}(E2)_{\mathrm{col}}. \tag{3}$$

The single-particle term has matrix elements between quasiparticle states

$$\langle 0|\alpha_j\mathfrak{M}_\mu(E2)_{\mathrm{s.p.}}\alpha_{j'}{}^\dagger|0\rangle = e_{\mathrm{eff}}\langle j|r^2Y_\mu{}^2|j'\rangle(U_jU_{j'}-V_jV_{j'}). \tag{4}$$

It also has matrix elements between quasiparticle states, each of which has, in addition, one phonon, but these may be ignored since they will always be overwhelmed by the collective matrix elements discussed below. The effective charge, e_{eff} of Eq. (4), which is to take into account the quadrupole polarization of the core by the particles of the last major shell which are used explicitly in the calculation, is chosen as $e_{\mathrm{eff}}=1$ for neutrons and $e_{\mathrm{eff}}=2$ for protons.

The collective term has matrix elements between states differing by one unit in the number of phonons present. The simplest such matrix element is related to the reduced $E2$ transition rate for exciting the first excited state of an even nucleus, i.e., the one-phonon state:

$$B(E2)_{0+\to2+}=\sum_{\mu m_f}|\langle 0|B_{m_f}\mathfrak{M}_\mu(E2)_{\mathrm{col}}|0\rangle|^2, \tag{5}$$

where $B(E2)$ is the usual reduced $E2$ transition probability. The collective matrix elements important for the odd-mass transitions are those between a wave-function component containing just a quasiparticle and one containing a quasiparticle and a phonon. Aside from a simple geometrical factor, these matrix elements are just the same as that given by Eq. (5). In order to utilize the information from the even nuclei as much as possible, in evaluating the above matrix element, the average of experimental $B(E2)_{0+\to2+}$ values from neighboring even-even nuclei is used rather than the expression derived in Ref. 4 for this quantity.

The final form for the reduced transition probability for an $E2$ transition between two states of the form of Eq. (2) becomes

$$\frac{B(E2)_{j_i\to j_f}{}^{\mathrm{theor}}}{2j_f+1}=\Bigg|C_{j_i00}{}^{j_i}C_{j_f00}{}^{j_f} \times e_{\mathrm{eff}}\frac{\langle f|r^2|i\rangle}{(4\pi)^{\frac12}}(-1)^{j_f-\frac12}C_{\frac12}{}^{j_i}{}_{-\frac12}{}^{j_f}{}_0{}^2(U_iU_f-V_iV_f) + \left[\frac{B(E2)_{0+\to2+}}{5}\right]^{\frac12}\Big[(-1)^{j_i-j_f}(2j_f+1)^{-\frac12}C_{j_f00}{}^{j_f}C_{j_f12}{}^{j_i} + (2j_i+1)^{-\frac12}C_{j_i12}{}^{j_f}C_{j_i00}{}^{j_i}\Big]\Bigg|^2. \tag{6}$$

The single-particle estimate with which to compare is

$$\frac{B(E2)_{j_i\to j_f}{}^{\mathrm{s.p.}}}{2j_f+1}=e^2\frac{\langle f|r^2|i\rangle^2}{4\pi}(C_{\frac12}{}^{j_i}{}_{-\frac12}{}^{j_f}{}_0{}^2)^2. \tag{7}$$

In Eqs. (6) and (7) the factor $2j_f+1$ is included to make the theoretical expression symmetric in j_i and j_f. The theoretical expression, Eq. (6), may then be compared with experimental $B(E2)$ values obtained from Coulomb excitation or lifetime measurements. In the latter case the $B(E2)$ value is related to the partial lifetime by

$$1/T_\gamma(E2)=(4\pi/75)(E^5/\hbar^6c^5)B(E2). \tag{8}$$

In Table I, the theoretical values, Eqs. (6) and (7), are given for various possible transitions between low-lying states whose wave functions are computed in Ref. 4 for spherical nuclei from Ni to Pb. The corresponding experimental value is included when available.

III. DISCUSSION

The theoretical results for $E2$ transitions in odd nuclei are seen to range from a small fraction of the single-particle rate to more than one hundred times single particle. The very large rates occur only for nuclei whose even neighbors have particularly large $B(E2)_{0+\rightarrow 2+}$ rates. The small rates are not so common, only about 30 of the 150 calculated cases being less than single particle. These cases occur only if the factor $(U_iU_f - V_iV_f)$ is quite small, corresponding to λ, the Fermi energy of pairing theory, being midway between the two single-particle energies in question. The exact isotope for which this occurs depends sensitively on the original choice of the single-particle energies. The experimental $B(E2)$ values for odd nuclei in this region also vary over a range of a factor of 1000. The agreement between theory and experiment is quite good, the large majority of the forty or so cases agreeing to within about a factor of 2 with the theoretical result.

This agreement shows that there is considerable truth in the picture of wave functions of odd spherical nuclei consisting of linear combinations of quasiparticles and quasiparticles coupled to phonons. Furthermore, the phonons have the same properties as those of the neighboring even nuclei, and the mixing coefficients of Eq. (2) may be computed as in Ref. 4. Further experimental investigation is desirable, and the calculated rates may serve as a guide to the expected rates. Fast cases might be used to investigate the phonon character of the wave functions in more detail. On the other hand, an observation of the $E2$ rate in cases for which a large retardation from the single particle rate is predicted might help to determine the validity of the single-particle energies used in the calculation and also to indicate possible wave-function components not included in Eq. (2).

ACKNOWLEDGMENT

The author wishes to thank Dr. L. Grodzins, who helped locate some of the experimental results.

PHYSICAL REVIEW VOLUME 133, NUMBER 2B 27 JANUARY 1964

Spin-23/2⁻ Isomer of Lu^{177}†

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(Received 3 September 1963)

Investigations of the decay of the three-particle state in Lu^{177} with spin $23/2^-$ performed with the crystal diffraction technique revealed evidence for three-particle states in Hf^{177} and rotational bands in Lu^{177} and in Hf^{177}. Levels with spins to 17/2 were found in the $K=7/2^+$ rotational band in Lu^{177} while the $K=7/2^-$ and $K=9/2^+$ bands in Hf^{177} were found to be excited up to spin 21/2 levels. From energy and intensity measurements of the cascade, crossover, and interband transitions, the values of a number of parameters pertinent to the collective model were derived. In particular, it was verified for each of the rotational bands that the quantity $(g_K - g_R)/Q_0$ was a constant within the experimental error.

INTRODUCTION

RECENTLY, Jorgensen *et al.*[1] have observed a 155-day isomeric state in Lu^{177} in a neutron-bombarded lutetium sample. From considerations of the decay mode they conclude[2] that this isomer has a very high spin of 23/2. Only a three-particle configuration of the odd proton of Lu^{177} and an uncoupled neutron pair could give rise to this high spin. In particular, the configuration obtained by adding the [624]$9/2^+$ neutron and the [514]$7/2^-$ neutron to the [404]$7/2^+$ proton is most likely to explain the observed Lu^{177} isomer.

The large change in the intrinsic configuration reduces the speed of the electromagnetic isomeric transition from the three-particle state so that it can compete with β decay into the neighboring Hf^{177}. In Hf^{177} similar three-particle states are expected to appear. A configuration based on the [514]$7/2^-$ neutron coupled to a [514]$9/2^-$ proton and a [404]$7/2^+$ proton could result in a state of spin $23/2^+$. Two other configurations favored by energy considerations both resulting in spin $21/2^+$ are obtained by coupling the [514]$7/2^-$ neutron to the [514]$9/2^-$ and [402]$5/2^+$ protons, or by coupling together three neutrons in [624]$9/2^+$, [514]$7/2^-$, and [512]$5/2^-$ orbits. Other combinations resulting in three-particle states of spin $25/2^-$, $19/2^-$, $17/2^+$, $15/2^+$, $15/2^-$, and $13/2^-$ can be constructed, but their respective energies are expected to be somewhat higher than those of the $23/2^+$ and $21/2^+$ configurations.

In this article we report on a study of the decay of the Lu^{177} isomer into Lu^{177} and Hf^{177}. Evidence for two three-particle states in Hf^{177} with spin $23/2^+$ and $21/2^+$

† Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] M. Jorgensen, O. B. Nielsen, and G. Sidenius, Phys. Letters **1**, 321 (1962).

[2] O. B. Nielsen (private communication).